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Importance of k_4 and $k_{2,3}$ in topological graph theory

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ABSTRACT: Outer planar graphs are a widely studied graph class with application in graph Drawing [T .Biedl, G. Kant] and with intersecting theoretical properties [S .Fesner, A.Maheshwari, J.Manning]. The outer thickness of the graph is the minimum number of outer planar into which the graph can be decomposed and outer thickness is one of the classical and standard measures of non- outer planarity of graphs. In this article we show that complete graph k_4 and bipartite graph $k_{2,3}$ is very important in graph theory and we suggest the rule of programming formulation of the outer thickness problem.

Keywords: Outer planar, outer thickness, k_4, $k_{2,3}$.

INTRODUCTION

A graph is usually visualized by representing each vertex through a point in the plane, and by representing each through a curve in the plane, connecting the points corresponding to the end vertices of the edge. Such a representation is called a during of the graph if no two vertices are represented by the same point, if the curve representing on edge not include any other points, representing a vertex (except its end points), and if two distinct edge have at most one common point (Porane, 2004).

A graph is outer planar if it admits a plane embedding where all its vertices surround the same region and no two distinct edge intersect, otherwise the graph is non- outer planar.

Characterization of outer planar graphs

Def (2.1). If a graph G' = *(V, E') is* a outer planar sub graph of G such that every graph *G"* obtained from *G'* by adding on edge from *E \ E'* is non- outer planar, then *G'* is called a maximal outer planar sub graph of G.

Def (2.2). Let G' = (V, E') be a maximal outer planar sub graph of G. if there is no outer planar sub graph *G" = (V, E") of G* with \E"\ >\ *E'\,* then G' is a maximum Outer planar sub graph.

Def (2.3). A graph H is said to be homeomorphic from G if either H= G or H is Isomorphic to a subdivision of G.

Theorem (2.1). A graph is outer planar if and only if it has no sub graph home orphic to K_4 or $K_{2,3}$

Theorem (2.2) A graph is outer planar if and only if G + K_1 is planar.

Theorem (2.3). [K.S.Kedlaya] Let G' = (V, E") be a maximum outer planar sub graph of a Graph G = *(V, E)* then $|E'| \le 2|V| - 3$.

Theorem (2.4). [K.S.Kedlaya] Let G' = (V, E') be a maximum outer planar sub graph of a graph G = (V, E) which does not contain any triangle. Then $|E'| \le 3|V|/_2 - 2$

Theorem (2.5). [R.K.Guy and et al] The maximum outer planar sub graph of Q_n contains $3\times 2^{n-1} - 2$ edges.

Outer thickness

Def (3.1). The outer thickness of a graph, devoted by *θ(G),* is the minimum number of outer planar sub graph in to which the graph can be decomposed.

Theorem (3.3). Let G = *(V, E) be* a graph with $|V| = n$ and $|E| = m$. Then

$$\theta_0(G) \ge \left\lceil \frac{m}{2n-3} \right\rceil$$

Proof. By theorem (2.3) , the denominator is maximum size of each outer planar sub graph. The pigeonhole principle the yields the inequality.

Theorem (3.2). Let G = *(V, E)* be a graph with *\V* = n and *\E* = *m. and* with no triangle. then

$$\theta_0(G) \geq \left\lceil \frac{m}{3n/2-2} \right\rceil$$

Proof. By theorem (2.4) , the denominator is maximum size of each outer planar sub graph. The pigeonhole principle the yields the inequality.

Theorem (3.3). [R.K.Guy and et al] For complete graphs, $\theta_0 = \left\lceil \frac{n+1}{4} \right\rceil$, except that θ_0 (K_7) = 3

Theorem (3.4). [R.K.Guy and et al] For complete bipartite graphs with *m ≤ n,*

$\theta_0(k_{m,n}) = \left\lceil \frac{mn}{2m+n-2} \right\rceil$

Theorem (3.5) [T.Poranen] for a graph with minimum degree δ and maximum degreeΔ, it holds that

$$\left\lceil \delta/4 \right\rceil \leq \theta_0(G) \leq \left\lceil \frac{\Delta}{2} \right\rceil$$

suggestion for integer programming formulation of the outer thickness

Now we give an integer programming formulation of the outer thickness problem. We are encouraged by the fact that polyhedral combinatory and branch and cut algorithms have been successfully applied to the maximum outer planar program. A similar article (Mutzel et al., 1988)

Since the outer thickness value for practical problem instances is relatively small, only a few more variable are needed.

The facet defining inequalities accruing in the integer programming formulation for the maximum outer planar sub graph problem are the main ingredients for this formulation. Consider a graph G= (V, E) with (V) =n and (E) = m and let t be an upper bound for the outer thickness of G. The task is to assign each edge lead to a sub graph le {1, 2,…, t} in such a way that all sub graphs are outer planar sub graph is minimized . we introduce edge-variables $y_{l,e}$ for L=1,2,….,t and e=1,2,…,m which indicate if edge airs assigned to layer l($y_{l,e}$) or not ($y_{l,e}$=0).

In addition, we need layer – variables x_l for L=1,…., t which indicate if layer l is required , i.e., the layer contains at least one edge.

The integer programming formulation of the outer thickness problem can now be stated as follows:

Min $\sum_{l=1}^{t} x_l$

s.t $\sum_{l=1}^{t} x_{l,e} = 1$ for e=1,2,…,m

$\sum_{Le\in F_L} Y_{l,e} \leq |F_L| - 1$ for all subdivision f_l of k_4 and $k_{2,3}$ on layer L ,

for all L=1,2,…,t

$X_L \geq y_{L,l}$

$y_{L,l} \leftarrow \{0,1\}$

$x_L \in \{0,1\}$

CONCULSION

in this article we show that complete graph and complete bipartite graph is very important in graph theory.in particular complete and complete bipartite graphs k_4 , $k_{2,3}$.

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